A Hierarchical Demand-Response Algorithm for Optimal Vehicle-to-Grid Coordination

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Abstract—We propose an algorithm to deal with the problem of decentralized coordination of charging/discharging of a large population of plug-in electric vehicles (PEVs). We introduce a framework in which the power grid is modeled as an undirected rooted tree. The root of the tree represents the generation/transmission side of the system and the leaves represent PEVs. Intermediate nodes represent congestible elements on the distribution side (e.g., transformers), which have a bound on the demand they can attend. In the proposed algorithm, the root generates a control signal based on the price per unit of power according to the demand for each time. Intermediate nodes modify the control signal according to the difference between the demand they take care of, and its capacity upper bound. PEVs update their charging/discharging strategies according to this pricing signal. Simulations demonstrate the algorithm performance for a particular example.

I. INTRODUCTION

The introduction of renewable energy generation into the power grid would constitute a major step towards sustainable development. However, the random nature of renewable resources along with their fast variability pose a significant challenge to power system operators, who may have to resort to ancillary generation plants in order to balance the power generation with the demand. Though ancillary generation plants are fast and reliable, they are also very expensive to operate, which creates additional costs on both users and utilities. To avoid the use of expensive generators, a large research effort has been devoted to developing new demand-response strategies. This finds special application in the control of plug-in electric vehicle (PEV) charging in a way that minimizes their impact on the power grid [1]. A novel idea, known as vehicle-to-grid (V2G), considers the possibility that the fleet of PEV can deliver power to the grid in order to provide ancillary generation service [2] by taking power when demand is low to deliver it when demand is higher. This is in contrast with the so-called V1G (or half V2G), in which power only flows from the power grid towards the fleet of PEVs, providing only demand response capacity. As useful as it sounds, the use of the PEV fleet for power supply must be also coordinated to prevent additional demand peaks that will further harm the system performance. A desired property of demand control is that of decentralization; that is, by which the decision making is carried out by the user side following a coordination signal that comes from the utility. This helps preserve user privacy, since no usage parameters are provided to the utility, and allows for scalability, since the computation approach is distributed among the loads. In this manuscript we study a decentralized V2G algorithm that serves this purpose.

A large body of literature has been devoted to PEV coordination. Most available works deal with the V1G idea in a centralized way, see [3], [4]. More recently, [5], [6] present decentralized solutions to the V1G problem. In the latter, a coordination signal is transmitted by the utility and it is used by the PEVs to choose their charging strategy by solving a local optimization problem. Their updated charging strategy is then fed back into an aggregator located on the utility side which computes a new coordination signal. In [7], a coordination signal is used for decentralized PEV charging control, but the charging strategy is computed using a load-balancing algorithm, which allows for convergence with less computational effort. In [8] the authors present a solution to the V1G problem in [6], by considering a distribution grid with some power capacity limitations. However, the algorithm is computationally intense, since many inner-loop iterations are run at each step. Simultaneously, coordination algorithms of large groups of loads has been presented in [9], [10], [11] respectively. The authors make use of several aggregators to handle different subsets of thermal loads that are used for demand response. For V2G, available works include [12], where a centralized optimization problem is solved using simulated annealing and ant-colony optimization. Further, [13] presents a purely centralized optimal control algorithm to solve a V2G problem accounting for uncertainty. An alternative approach that follows a game-theoretic formulation is given in [14], where PEVs are modeled as batteries that inject to or draw from an aggregator a certain amount of energy in order to meet a desired aggregate energy level. The previous work does not focus on load shifting, but it computes a memoryless solution of a game given an instantaneous demand value from the aggregator. None of the decentralized approaches considers power flow constraints, since in general, solution approaches to optimal power flow, e.g., [15], are completely centralized.
The present work introduces a decentralized approach for the coordination of a V2G protocol in a fleet of PEVs distributed over the power grid. We model the power grid/energy market as a tree network in which the root represents the generation/transmission side of the grid, and all other nodes represent either components in the distribution side or PEVs. Each component on the distribution side has a non-PEV load that must be satisfied and presents limitation in the amount of power that can deliver. In our decentralized algorithm all PEVs solve a local optimization problem to compute their charging/discharging profile for a finite discrete-time horizon. Then, they communicate it to the node at which they are connected, which in turn aggregates its PEV load, along with its non-PEV load and sends it to the node from which it draws power. The aggregation is performed in a cascaded way until the overall load reaches the root node which uses such information to compute a control signal that is sent back to all nodes that are connected to it. Intermediate nodes modify the signal according to whether or not their maximum power capacity is violated. To this end, we use penalty functions associated to these constraints. Next, the modified signal is transmitted down into the tree network, again in a cascaded way until it reaches all the PEVs. Then, PEVs use this signal to solve their local optimization problem again and repeat the cycle. In our approach, almost the entire computational load falls on the PEVs, while all other nodes in the network only act as aggregators. This results into good scalability properties of the algorithm. A convergence analysis is performed and simulations show the algorithm performance on a particular case.

A. Preliminary Notation

Let $\|x\|$ represent the Euclidean norm of $x \in \mathbb{R}^n$. For a finite set $\mathcal{A}$, $|\mathcal{A}|$ represents the number of elements in $\mathcal{A}$. If $\mathcal{B}$ is also a finite set, $\mathcal{A} \setminus \mathcal{B} = \{a \in \mathcal{A} | a \notin \mathcal{B}\}$. For $x \in \mathbb{R}$, $|x|$ denotes the absolute value of $x$.

B. On graph theory

Consider an undirected graph $\mathcal{G} \doteq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of edges. A path $\mathcal{P}(i, j)$, on $\mathcal{G}$ for nodes $j, j \in \mathcal{V}$, is defined as a sequence of nodes $\{n_1, \ldots, n_r\}$ such that $n_1 = i$, $n_r = j$, and $(n_{\ell}, n_{\ell+1})$ is an edge of $\mathcal{G}$, for all $\ell \in \{1, \ldots, r-1\}$. $\mathcal{G}$ is a tree if there is a unique path between any two nodes $i, j \in \mathcal{V}$. For an undirected graph, any $r \in \mathcal{V}$ can be called a root of $\mathcal{G}$. Then, $\mathcal{G}$ is a rooted tree and there is a partial order of all nodes in $\mathcal{V}$ according to their distance to $r$, where the distance between two nodes is defined as the number of links on the path connecting them. For a node $j \in \mathcal{V}$, the set of children $\text{ch}(j)$ is comprised by all nodes that are connected by one link to $j$, and whose distance to $r$ is equal to the distance from $j$ to $r$ plus one. The set of descendants of $j$, $\text{des}(j)$, is the set of all nodes $i \in \mathcal{V}$ such that $j \in \mathcal{P}(r, i)$, and the distance from $r$ to $i$ is larger than the distance from $r$ to $j$. Similarly, the parent of $j$, denoted as $\text{pr}(j)$ is the node connected to $j$, whose distance to $r$ is equal to the distance from $r$ to $j$ minus one. The set of ancestors of $j$, denoted as $\text{an}(j)$, is the set of all nodes in $\mathcal{P}(r, j) \setminus \{j\}$. A node $j \in \mathcal{V}$ is called a leaf of $\mathcal{G}$ if it is only connected to one node $l \in \mathcal{V}$.

II. Problem Formulation

Consider a population of $n$ plug-in electric vehicles (PEV) that is connected to the power grid. This population is spread over a large area. The objective of each PEV is to fully charge its battery before a user-defined deadline. Each electric vehicle is also able to deliver power to the grid. Let us think of the power grid as a hierarchical structure in which power flows from generators toward users through several different layers: substations, transformers, etc. Each of these elements can be seen as congestible (which can be congested) due to the limit on the amount of power they can deliver.

A. Structure of the power network

We consider a network where a single node abstracts away the generation capacity, along with the energy market, and the transmission side of the power grid, which provides energy for all users and computes a price-per-unit based on the supply and demand. From this node, the power branches out to distribution units, and at those units, there is a further branching towards smaller distribution nodes, until it reaches the end user.

This model can be mathematically described as an undirected rooted tree $T \doteq (\mathcal{V}, \mathcal{E})$, where all PEVs correspond to leaves of the tree graph, i.e., nodes with a single neighbor. Without loss of generality, let $\mathcal{N} \doteq \{1, \ldots, n\}$ be the set of PEVs in the grid. In addition, let the set $\mathcal{M} \doteq \{n+1, \ldots, r\} = \mathcal{V} \setminus \mathcal{N}$, $r \notin |\mathcal{V}|$, be the set of all nodes that are not PEVs, namely the set of congestible elements in the grid. These nodes have an associated non-PEV demand that must be attended. This demand comes from a forecast process and it is know only to the corresponding node. Let the node $r \in \mathcal{M}$ be the generation/pricing node, i.e., the root of the tree graph. Since the leaves represent only PEVs, these nodes do not have any non-PEV load associated to them.

Remark 2.1: The tree topology is a reasonable assumption, given that the distribution side of the grid generally follows a radial structure. Most existing distribution feeders can be described by this framework. In Figure 1 it can be seen that the tree topology is the foundation of the communication protocol we define for the proposed algorithm in Subsection III-B.

For each leaf node $i$ in the tree graph, i.e., PEVs, the set $\text{ch}(i) = \emptyset$, while for the node $r$, $\text{pr}(r) = \emptyset$; see a particular example topology in Figure 1. Furthermore, for each node $j$ in the set $\mathcal{M}$, there is a parameter $P_j^{\max}$ that corresponds to an upper bound on the amount of power that the node can provide its children and its own non-PEV load during a time slot $t \in \mathbb{N}$.
Fig. 1. Grid hierarchical structure. Dashed lines indicate communication links while solid lines represent power links.

B. PEV battery model

We assume that the battery of each PEV follows the dynamics:

\[ z_{i,t+1} = z_{i,t} + \frac{\alpha_i^c}{\beta_i} u_{i,t} - \frac{1}{\alpha_i^d} v_{i,t}, \]

where \( u_{i,t} \geq 0 \) is the amount of energy that is charged into the battery during time interval \( t \in \mathbb{N} \), \( v_{i,t} \geq 0 \) is the energy discharged from the battery during time interval \( t \in \mathbb{N} \), \( \alpha_i^c \in (0,1) \) is the battery system charging efficiency, \( \alpha_i^d \in (0,1) \) is the battery system discharging efficiency, \( \beta_i \) stands for the battery capacity, and \( z_{i,t} \) is the state of charge (SOC) at time \( t \in \mathbb{N} \). The SOC must satisfy \( z_{i,t} \in [z_{i,\min}, z_{i,\max}] \), for \( 0 \leq z_{i,\min} < z_{i,\max} \leq 1 \). In addition, some power bounds must be established in the battery charging/discharging, namely \( u_{i,t} \leq u_{i,\max} \) and \( v_{i,t} \leq v_{i,\max} \). It is also reasonable to assume that the battery cannot charge and discharge at the same time, then it must also hold that \( u_{i,t} v_{i,t} = 0 \), for all \( t \in \mathbb{N} \). Then, each PEV, \( i \in \mathcal{N} \), can be characterized by a demand profile \( d_i = \{d_{i,t}\}_{t \in \mathbb{N}} \), where \( d_{i,t} = u_{i,t} - v_{i,t} \). Whenever the demand is negative, the PEV owner must be paid by the utility a price for its energy that is determined by the energy market, i.e., depends on the supply and demand.

C. Congestible elements

Each of the nodes \( i \in \mathcal{M} \) is considered a congestible element, which is characterized by: i) a demand profile \( d_i = \{d_{i,t}\}_{t \in \mathbb{N}} \) and ii) a maximum amount of power that can deliver. The demand \( d_{i,t} \) is given by the aggregate of the children’s demand, and the non-PEV demand associated to \( i \) at time \( t \), denoted by \( L_{i,t} \), i.e.:

\[ d_{i,t} = \sum_{\ell \in \text{ch}(i)} d_{\ell,t} + L_{i,t}. \]

For a further characterization, define the set \( dN(j) = \mathcal{N} \cap \text{des}(j) \). Thus, we can rewrite (1) as:

\[ d_{i,t} = \sum_{\ell \in \text{des}(i) \cap \mathcal{M}} d_{\ell,t} \quad \forall t \in \mathbb{N} \]

for all \( i \in \mathcal{M} \). Note that the first sum corresponds to the entire non-PEV load of the descendants of \( i \) that are not PEVs, the second term corresponds to the demand by all the PEVs that are descendants of \( i \), and the third term is plainly the non-PEV demand of node \( i \). Recall that PEV nodes correspond to the power outlet for the PEV only, then they do not have non-PEV load. Given the maximum amount of power \( i \) can deliver and the length of each time slot \( t \), we denote by \( P_i^{\text{max}} \) the maximum power that \( i \) can provide during at a time slot \( t \).

D. The generation/pricing node

The node \( r \in \mathcal{V} \), referred to as the generation/price node models the behavior of the generation/transmission side of the power grid. For simplicity, we consider that this node can provide a maximum amount of power \( P_r^{\text{max}} \), and the cost of providing a constant amount of power \( x \) during the entire time slot \( t \in \mathbb{N} \) is given by \( C(x) \), where \( C : \mathbb{R} \to \mathbb{R}_{>0} \) is a convex and increasing function, and \( C(0) = 0 \). This function also determines the price that will be paid to the owner of the \( i \)-th PEV when it delivers power into the grid, i.e., \( C(v_{i,t}) \).

Assumption 2.1 (Derivative of \( C \) is Lipschitz): The function \( C \) is such that \( C' \) is Lipschitz in its domain, with Lipschitz constant \( l_C \).

III. A decentralized control architecture

In order to compute the charging/discharging profile for the fleet of PEVs getting energy from the power grid, we propose a finite-horizon discrete-time optimal control problem. Our main interest is to introduce a decentralized communication and control architecture that allows distributed computation, scalability, and privacy.

A. Optimal control problem

The charging coordination strategy must be able to minimize the cost function corresponding to the total cost of the energy provided by the utility during a finite horizon \( \tau = \{1, \ldots, T\} \), while respecting the power constraints in all the congestible elements of the grid.

Problem 1: \( \min_{u,v} J(u,v) \)

subject to:

\[ (u_{i,t}, v_{i,t}) \in \mathcal{F}_i, \quad \forall i \in \mathcal{N} \]  
\[ d_{i,t} = u_{i,t} - v_{i,t}, \quad \forall t, i \in \mathcal{N} \]  
\[ d_{j,t} = \sum_{\ell \in \text{ch}(j)} d_{\ell,t} + L_{j,t}, \quad \forall t, j \in \mathcal{M} \]  
\[ d_{j,t} \leq P_j^{\text{max}}, \quad \forall t, j \in \mathcal{M}, \]
where:
\[ J(u, v) = \sum_{t=1}^{T} C(d_{t,v}), \]
and \((u_i, v_i) \in F_i\) if the following constraints hold:
\[
z_{i,t} = z_{i,0} + \frac{1}{\alpha_i} \sum_{\ell=1}^{T} \left( \alpha_i^\ell u_{i,\ell} - \frac{1}{\alpha_i^\ell} v_{i,\ell} \right), \quad t \in \tau, \] (5a)
\[
z_{i,\min} \leq z_{i,t} \leq z_{i,\max}, \quad t \in \tau, \] (5b)
\[
0 \leq u_{i,t} \leq u_{i,\max}, \quad t \in \tau, \] (5c)
\[
0 \leq v_{i,t} \leq v_{i,\max}, \quad t \in \tau, \] (5d)
\[
u_{i,t} = 0, \quad t \notin Z_i, \subseteq \tau, \] (5e)
\[
u_{i,t} = 0, \quad t \notin Z_i, \subseteq \tau, \] (5f)
\[
z_{i,T} = z_{i,\max}, \] (5g)
\[
u_{i,t} \nu_{i,t} = 0, \quad t \in \tau. \] (5h)

Notice that the constraint (5h) is not convex. The following result allows us to relax the constraint without affecting the solution of the problem.

**Lemma 3.1 (Exact convex relaxation):** The constraint \(u_{i,t} \nu_{i,t} = 0, \) for all \( t \in \tau, \ i \in \mathcal{N}\) can be relaxed and the optimal solutions for Problem 1 are exactly the optimal solutions for Problem 1.

**Proof:** It can be easily shown that if \(u^*, v^*\) is an optimizer of Problem 1 and there is some \( i \in \mathcal{N}\) and \( t \) such that \(u^*_{i,t} > 0\) and \(v^*_{i,t} > 0\), there exists a feasible solution \(\hat{u}, \hat{v}\) such that \((\hat{u}_{j,q}, \hat{v}_{j,q}) = (u^*_{j,q}, v^*_{j,q})\) for all \((j,q) \neq (i,t), t, q \in \tau, j \in \mathcal{N}, \) and \(\hat{u}_{i,t} = \max\{0, u^*_{i,t} - (\alpha_i^\ell)\}, \) and \(\hat{v}_{i,t} = \max\{0, v^*_{i,t} - (\alpha_i^\ell)\}\) such that \(\sum_{j \in \mathcal{N}} (\hat{u}_{j,t} - \hat{v}_{j,t}) < \sum_{j \in \mathcal{N}} (u^*_{j,t} - v^*_{j,t}),\) and \(\sum_{j \in \mathcal{N}} (\hat{u}_{j,t} - \hat{v}_{j,t}) = \sum_{j \in \mathcal{N}} (u^*_{j,t} - v^*_{j,t}),\) for all \(q \neq t, q \in \tau.\) Given that the function \(\Phi\) is convex and increasing, \(J(\hat{u}, \hat{v}) < J(u^*, v^*),\) which contradicts the fact that \(u^*, v^*\) is optimal.

The next result is an adaptation of Theorem 1 in [6], and shows the uniqueness of the optimal demand profile generated by the optimizers of Problem 1.

**Lemma 3.2: (Uniqueness of the aggregate demand profile):** Let \(u^*, v^*\) and \(\hat{u}, \hat{v}\) be optimizers of Problem 1. Then it holds that \(\sum_{i \in \mathcal{N}} (u^*_i - v^*_i) = \sum_{i \in \mathcal{N}} (\hat{u}_i - \hat{v}_i).\)

The solution of this optimization problem is valley filling and peak-shaving, i.e., if \(u, v\) is optimal, the PEVs will try to provide as much energy as possible in the highest-price times and will try to obtain as much energy as possible in the lowest-price times.

**Remark 3.1:** Notice that since the net demand in a node \(j \in \mathcal{M}\) can be negative, there could be a lower-bound constraint on it. We do not consider such constraint in this study, the problem formulation and the analysis can be trivially extended to it. In addition, this framework can be easily extended to a case in which \(P_{j}\) varies in time. In [8] the authors mention the case of a transformer, which requires a cooling time, during which its maximal power capacity may decrease.

In order to solve Problem 1, we use penalty functions to handle the coupling constraints. We formulate the following relaxation of the problem:

**Problem 2:**
\[
\min_{u,v} J(u,v) + \sum_{t=1}^{T} \sum_{\ell \in M} \kappa_{\ell} \Phi_{\ell}(d_{\ell,t})
\]
subject to:
\[
(u_i, v_i) \in F_i, \quad \forall i \in \mathcal{N} \quad (6a)
\]
\[
d_{i,t} = u_{i,t} - v_{i,t}, \quad \forall t, i \in \mathcal{N} \quad (6b)
\]
\[
d_{j,t} = \sum_{\ell \in \mathcal{J}(j)} (d_{\ell,t} + L_{j,t}), \quad \forall t, j \in \mathcal{M} \quad (6c)
\]
where \(\Phi_{\ell}: \mathbb{R} \to \mathbb{R}_{\geq 0}\) acts as a penalty function for the power constraint in node \(j \in \mathcal{M},\) defined as:
\[
\Phi_{\ell}(d_{\ell,t}) = \left(\max\{0, d_{\ell,t} - P_{j}^{\max}\}\right)^2,
\]
and \(\mathcal{F}_{i} = \{(u_i, v_i) \in \mathbb{R}^{2nT} | \text{Eqns. (5a)--(5g) hold}, \forall i \in \mathcal{N}\}.\) Notice that:
\[
\Phi'_{\ell}(d_{\ell,t}) = \max\{0, 2(d_{\ell,t} - P_{j}^{\max})\},
\]
then, \(\Phi'_{\ell}(d_{\ell,t})\) is globally Lipschitz continuous with Lipschitz constant \(l_{B} = 2,\) for all \(j \in \mathcal{M}.\)

**Remark 3.2:** The penalty method leads to a solution of the optimal control problem for \(\kappa_{\ell} \to +\infty, \ell \in \mathcal{M}.\) A way to deal with this problem is to choose the parameter \(P_{j}^{\max}\) slightly lower than the physical limit of the node \(j\) and choose \(\kappa_{\ell}\) large enough. It guarantees that the solution of Problem 2 is feasible for Problem 1 and close enough to an optimizer of it.

Next, we introduce a decentralized approach for the solution of Problem 2. For now on, let us assume that the optimization problem is feasible.

**B. Decentralized V2G coordination algorithm**

Our approach endows each congestible element in the grid with computation and communication capacity. Communications over the network follow the same tree topology as the power network. Thus, each element \(i \in \mathcal{V}\) sends its parent the demand profile \(d_i\) and if \(i \in \mathcal{M},\) implying that \(\text{ch}(i) \neq \emptyset, \) it sends its children a control signal which comes from the generation/pricing node \(r,\) and the violation on the maximum energy constraint.

The algorithm is inspired by the works presented in both [5], [6] but we modify the approach to account for the penalty functions of Problem 2. This is an iterative procedure in which at iteration \(k \in \mathbb{N},\) each PEV generates a demand profile \(d_{i,k} = u_{i,k} - v_{i,k},\) for all \(t, i, \tau,\) that is feasible for its own battery constraints. Then, it transmits the profile to its parent, which in turn computes its own demand profile according to Equation (1). This is done until the node \(r\) computes its demand profile; see Figure 1 for an illustration of the information flow over the communication network.

Based on this demand profile, \(r\) provides a coordina-
tion signal \( p_{k}^{+} = [p_{r,1}^{k}, \ldots, p_{r,T}^{k}]^{T} \in \mathbb{R}^{T} \), such that:

\[
p_{r,t}^{k} = \eta C'(d_{r,t}^{k}),
\]

for all \( t \in \tau \) and some \( \eta > 0 \). Then, each node \( j \in \mathcal{M} \setminus \{r\} \) computes the control signal \( p_{j}^{k} = [p_{j,1}^{k}, \ldots, p_{j,T}^{k}]^{T} \in \mathbb{R}^{T} \):

\[
p_{j,t}^{k} = p_{pr(j),t}^{k} + b_{j,t}^{k},
\]

for all \( t \in \tau \), where:

\[
b_{j,t}^{k} = \eta \kappa_{j} d_{j,t}^{k},
\]

and some \( \kappa_{j} > 0 \). Then, each PEV computes its next battery control by solving the optimization problem:

\[
(u_{i}^{k+1}, v_{i}^{k+1}) = \arg\min_{u_{i},v_{i}} J_{i}(u_{i}, v_{i})
\]

subject to:

\[
(u_{i}, v_{i}) \in \mathcal{F}_{i},
\]

where:

\[
J_{i}(u_{i}, v_{i}) = \sum_{t=1}^{T} p_{pr(i),t}^{k}(u_{i,t} - v_{i,t}) + \frac{1}{2}\|u_{i} - v_{i} - d_{i}^{k}\|^{2}.
\]

The procedure must be iterated until some stopping criterion is reached.

**Theorem 3.1 (Convergence result):** The algorithm converges to an optimizer of Problem 2 as \( k \to \infty \), provided Assumption 2.1 (Derivative of \( C \) is Lipschitz) holds, \( \eta < \min\{\{nC(1 + |\alpha(i)|)^{-1} \mid i \in \mathcal{N}\}, \eta < \min\{\{\kappa_{d,1} d_{N}(i)|\alpha(\ell)|(1 + |\alpha(\ell)|)^{-1} \mid i \in \mathcal{N}, \ell \in \alpha(i)\}\} \). The proof of this result may be found in the extended version of this manuscript [16].

**IV. Simulations and discussion**

Our simulation scenario consists of \( N = 150 \) PEVs attached to a structure with 5 nodes, as shown in Figure 2. The root node, i.e., \( N + 5 \) is not assigned any PEV. All PEV battery capacities, efficiency parameters, deadlines, and parents have been randomly generated, individually for each PEV. The time-horizon contains 48 slots with a duration of 30 minutes each. We establish bounds for the power to be drawn from nodes \( N + 1 \) and \( N + 2 \) as: \( P_{21}^{\max} = 56 \) and \( P_{22}^{\max} = 101.5 \). For the penalty functions associated to these constraints, we have chosen \( \kappa_{21} = \kappa_{22} = 1000 \), and \( \eta = 75 \times 10^{-5} \). There is no bound for the power to be drawn from nodes \( N + 3 \) through \( N + 5 \). The function \( C \) is chosen to be \( C(x) = x^{2} \). This function is increasing in the domain, since for our scenario its argument, i.e., the aggregate load is always nonnegative. All non-PEV demand, as well as the initial conditions, efficiency and battery capacities can be found at [http://fausto.dynamic.ucsd.edu/andres](http://fausto.dynamic.ucsd.edu/andres).

Figure 3 shows the optimal aggregate demand for all the PEVs in \( \mathcal{N} \), for a centralized solution of the exact problem, i.e., without penalty functions (red curve), and the aggregate demand given by the decentralized algorithm after 5000 iterations (black curve). It can be observed that the decentralized solution almost matches the aggregate given by the centralized benchmark. Figure (4) show the aggregate PEV and non-PEV demand for the centralized solution (red) and for the decentralized solution (black). In addition, we show in blue the non-PEV demand. It can be seen that the optimal solution is peak-shaving and valley-filling, since PEVs tend to provide energy between 12:00 and 17:00, and they charge between 22:00 and 8:00. Figures 5 and 6 show the demand curve for the nodes \( N + 1 \) and \( N + 2 \) respectively, for both centralized and decentralized cases. In green we show the upper bound on the power that each node can provide.

**V. Conclusions**

We present a decentralized protocol for a vehicle-2-grid (V2G) system in which a fleet of plug-in electric vehicles must coordinate their charging/discharging strategies to minimize a cost function consisting in the price of the total energy provided by the utility during a finite discrete-time horizon. The power flow leaves the transmission side of the power grid and enters the distribution side. It is modeled as a rooted tree, where nodes represent congestion elements (substations, transformers, etc.) which have a limited power delivery capacity. In order to account for these constraints, we use penalty functions. The algorithm does not require communication between PEVs, and the coordination signal is transmitted from the utility in a cascading way through the tree network, where it gets modified at each non-PEV node,
until it gets to the PEVs. Then, each PEV updates its charging/discharging profile. The algorithm converges to the optimizer of the cost function given the network constraints. Simulations show the system behavior.

As a future direction, we aim to address the power constraints in the congestible elements using non-differentiable penalty functions that allow exact solutions of the original problem, but require a subgradient-based algorithm for the solution, with the ensuing complications in the analysis.

References


